Program : Program : B.Sc. Code (BScGen03)   **Semester : VI**
Program (Specific) : Mathematics   **Course Type: Core**
Class : T.Y. B. Sc. (Regular)   **Max. Marks : 35**
Name of the Course : Real Analysis II   **Course Code : MT-362**
Time : 2 Hrs.   **SET A**

Instructions to the candidate:

1) *Figures to the right indicate full marks.*
2) *Draw a well labeled diagram wherever necessary.*

Solve any SEVEN of the following.   **[7 x 5 = 35]**

1) Define   (a) Refinement of [a , b]   (b) Upper sum of a function

     (c) Riemann Integrable function   (d) First fundamental theorem of

     calculus   (e) Abel's Test.

2) Prove that constant function is Riemann integrable on [a , b] $\subset \mathbb{R}$.

3) For n $\in$ I , let σ_n be the subdivision $\{ \ 0 \ , \frac{1}{n} , \frac{2}{n} , \ \ldots\ldots\ldots , \frac{n}{n} = 1 \ \}$ of

   [0 , 1]. Compute $\lim\limits_{n \to \infty} U[f ; \sigma_n]$ for the function f(x) = x².

4) Discuss the convergence of $\int_0^1 \frac{1}{\sqrt{x}}$ dx.

5) Discuss the convergence of $\int_0^2 \frac{x}{1-x}$ dx.

6) Show that $\int_0^1 x^{m-1}(1-x)^{n-1}$ dx is convergent if and only if m

   and n are both positive integers.

7) Prove that $\int_0^\infty x^{m-1}e^{-x}$ dx is convergent if and only if m > 0.

8) State and prove Cauchy criterion for uniform convergence of a

   sequence of real valued functions.

9) If $f_n(x) = \dfrac{n}{x+n}$, $x \geq 0$ then show that $\{f_n(x)\}_{n=1}^{\infty}$ is uniformly

   convergent on any finite interval $[0 , a]$, for $a \in \mathbb{R}$.

10) Discuss the convergence of the series $\sum_{n=0}^{\infty} x^n$, $x \in \mathbb{R}$.